IB Paper 4: Fluid Dynamics
Hints on Examples Paper 4

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1. Review your IA Dimensional Analysis notes. The scaling arguments require knowledge of "typical values" (see Lectures §5.5) for the dynamic and hydrostatic pressures: ρV^2 and $\rho g h$ respectively. These need to be multiplied by an area, or something proportional to area, to get a force.

3. Dimensional analysis for this problem shows that in general $C_D^{\text{total}} = f(Fr, Re)$, and under the given assumptions, we can make this relation more specific as $C_D^{\text{total}} = g(Fr) + kRe^{-1/5}$. Further, we know that the contributions of both terms are equal, and measure the total drag force, at model conditions. This information is sufficient to solve for k and evaluate the equation at real conditions with a different Reynolds number.

4. From the definition of total pressure (Lectures §7.1) in the reservoir $p_\text{t} = p_\text{atm}$, at the top of the fountain $p_\text{t} = p_\text{atm} + \rho g h$, and at the bottom of the fountain $p_\text{t} = p_\text{atm} + \frac{1}{2}\rho V^2$. Assuming no loss in the fountain gives an expression for the unknown V. The pump pressure rise must balance the change in total pressure from the reservoir to the fountain and pressure loss due to friction in the pipe (Lectures §7.2). Power is volume flow rate times total pressure rise (Lectures §7.6). In part (b), the velocity in the pipe is not the same as at the bottom of the fountain, and must be found from continuity.

6. Similar to question (4), we can use the definition of total pressure to directly calculate the change between reservoirs, and balance this against losses or gains in the system. In particular, between two reservoirs where the velocity is negligible, $\Delta p_\text{t} = \rho g \Delta h$. Balancing the total pressure change up to both high reservoirs gives two simultaneous equations in V_2 and V_3 which can be combined to form the part (a) result.

7. Refer to Lectures §7.6 for a discussion of mechanical energy in this context. Total pressure is like an energy per unit volume, so multiplication by volume flow rate gives an energy flow. What the question is after is the net $\sum_i Q_i p_{\text{t}\,i}$ for the i streams entering or leaving the control volume.

8. To achieve the same flow rate, the pipe exit velocities need to be the same. As the system is flat and under a uniform atmospheric pressure, this translates into equal total pressure at all exits. The job of the valves is to introduce additional pressure loss to satisfy this requirement. For part (c), consider what is constant and what is variable in your expression for pump pressure rise.

S3. The pressure gradient across the bend is set by the bulk main-stream flow and imposed on the relatively small boundary layer. For curving streamlines, $\partial p/\partial n = \rho V^2/\mathcal{R}$ is always true. Consider how moving into the boundary layer might affect the quantities in this equation.

9. The analysis is similar but not identical to Lectures §9.3, as we have to allow for the fact that the streamline of interest is not necessarily coincident with the edge of the boundary layer.

10., 11. These are amply addressed in Lecture 10. Do you think it is a coincidence that the Reynolds numbers for most ball sports are in such a narrow range?