IB Paper 4: Heat Transfer
Hints on Examples Paper 5

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S1., 1. Before starting questions like this, it is vital to draw a large, clear, labelled diagram of the situation. Treat as a set of thermal resistors in series; values for conduction and convection in plane geometries are given in Lectures §1.1 and 1.3, and conduction in a cylindrical geometry in Lectures §1.5.

2. Write an expression for the total thermal resistance, conductive plus convective, as a function of the lagging thickness; the worst possible result (maximum heat loss) corresponds to the minimum thermal resistance. Then, equate the thermal resistance without lagging to that with lagging to find the limiting thickness beyond which there is a benefit. The equation has no analytical solution so must be solved numerically. The final part is a repeat with a different radius parameter.

3. The Biot number $Bi = h\mathcal{V}/\lambda A$ is the ratio of internal conductive to surface convective thermal resistances. The lumped heat capacity model, where temperature of the body is uniform, is valid for low $Bi \ll 1$. If the temperature of the body very non-uniform due to a high conductive resistance, we have high $Bi \gg 1$. Solutions of the heat equation in these limiting cases are in Lectures §2.1–2.2.

4., 5. The maximum possible heat transfer is $(\dot{m}c_p)_{\min}(T_{\text{hot,in}} - T_{\text{cold,in}})$, the denominator of heat exchanger effectiveness. The overall heat transfer coefficient U is another way of expressing the total thermal resistance $R_{T,\text{total}} = 1/UA$ with a specified reference area A. The log-mean temperature difference is given in the Data Book; each ΔT are defined at a point in physical space not for the hot or cold fluid. To determine the variation in temperature along the tubes, apply the First Law to a differential element, see Lectures §3.2.

6. First look up fluid properties at the film temperature in the Data Book. This is a rough approximation because the properties vary continuously between the plate and main-stream temperatures. The Reynolds analogy is also in the Data Book, so we can determine h from c_f, and hence $\dot{q}$. The Prandtl number is the ratio of momentum to thermal diffusivities, $Pr = \mu c_p/\lambda$. We can use this to correct the Reynolds analogy when $Pr \neq 1$: from experiments $St = \frac{1}{2}c_f Pr^{-2/3}$, see lectures §4.6.

7. Dimensional analysis tells us that $Nu = \text{fn}(Re, Pr)$. So for similarity, we need to match Re and Pr between the benzene and water cases. Set T first followed by V, because $Pr = \text{fn}(T)$ only but $Re = \text{fn}(T, V)$. Because we have ensured similarity, we can then equate the $Nu = hL/\lambda$ in each case to evaluate the real h.

8. Remember that the Grashof number is analogous to the Reynolds number for forced convection, and think about what the Reynolds number characterises. As the first step, list fluid properties at the film temperature. Recall that the coefficient of expansion $\beta = \partial v/\partial T|_p/v$, and we already know an equation of the form $v = \text{fn}(p, v)$ for an ideal gas. We now have enough information to calculate Gr and evaluate the correct correlation for Nu.

9. As always, get the fluid properties at the film temperature first. Write an expression for $Re_x = \text{fn}(x)$ and substitute into the correlation to get an expression for $Nu_x(x)$. Then the definition of Nusselt number can be rearranged to give $h(x)$, and $\dot{q} = h(x)(T_\text{w}(x) - T_\infty)$ then gives $T_\text{w}(x)$ for the known uniform $\dot{q}$. Finally, the expression for $T_\text{w}(x)$ can be evaluated at $x = L$ and integrated over $0 < x < L$ to give the requested temperatures.