

3A5 - Basics, Availability and Exergy

THERMODYNAMIC PRINCIPLES

* Types of System

→ Isolated: constant mass and energy

→ Closed: constant mass, variable energy by $\dot{Q} \neq 0, \dot{W}_n \neq 0$

→ Open: variable mass, variable energy by $\dot{Q} \neq 0, \dot{W}_n \neq 0, \dot{m} \neq 0$

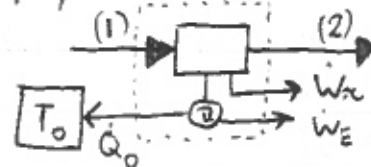
* "Simple compressible system" - only contribution to system energy is internal energy, KE, GPE etc. negligible.

* Types of Property

→ Extensive - proportional to system mass, e.g. V, U, H, S

→ Intensive - independent of system mass, e.g. p, T

→ Specific/Molar - extensive properties per unit mass/mole, also intensive



AVAILABLE WORK

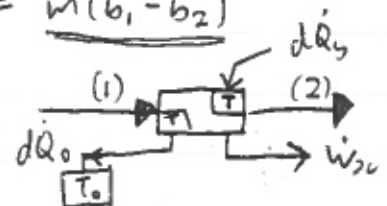
* Consider steady flow process from states (1) → (2), environment at T_0

* 1st Law on dotted CV $\Rightarrow \dot{m}(h_2 - h_1) = -\dot{Q}_0 - (\dot{W}_{sc} + \dot{W}_E)$

* 2nd Law on dotted CV $\Rightarrow \dot{m}(s_2 - s_1) = -\dot{Q}_0/T_0 + \dot{S}_{inrev}$

→ Eliminate $\dot{Q}_0 \Rightarrow \dot{W}_{TOT} = \dot{m}[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)] - T_0 \dot{S}_{inrev}$

→ $\dot{S}_{inrev} \geq 0$, so if $b = h - T_0 s$, $\dot{W}_{MAX} = \dot{m}(b_1 - b_2)$



ENERGY EQUATION: energy $e = b - b_0$

* D is dead (environment) state, $e_0 = 0$

* Consider steady flow process (1) → (2) with heat source and loss

* 1st Law on process $\Rightarrow \dot{m}(h_2 - h_1) = \int d\dot{Q}_s - \int d\dot{Q}_0 - \dot{W}_{sc}$

* 2nd Law on process $\Rightarrow \dot{m}(s_2 - s_1) = \int \frac{d\dot{Q}_s}{T} - \int \frac{d\dot{Q}_0}{T} + \dot{S}_{inrev}$

→ Combine: $\dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)] = \int (1 - \frac{T_0}{T}) d\dot{Q}_s - \dot{W}_{sc} - T_0 \dot{S}_{inrev} - \int (1 - \frac{T_0}{T}) d\dot{Q}_0$

→ Or $\dot{m}(e_2 - e_1) = \dot{E}_Q - \dot{W}_{sc} - \dot{W}_{L,CR} - \dot{W}_{L,Q}$

* \dot{E}_Q is "power potential" of heat addition = $\int (1 - \frac{T_0}{T}) d\dot{Q}_s$

* $\dot{W}_{L,CR}$ is lost work due to irreversibilities = $T_0 \dot{S}_{inrev}$

* $\dot{W}_{L,Q}$ is lost work due to environment heat transfer = $\int (1 - \frac{T_0}{T}) d\dot{Q}_0$

* Useful trick: $d\dot{Q} = \dot{m}dh = \dot{m}(Tds + vdp) = \dot{m}Tds$ if $dp=0$

3A5 - Gibbs Function, Chemical Reactions, etc.

GIBBS AND HELMHOLTZ FUNCTIONS - derived properties

* Gibbs $G = H - TS$, Helmholtz $F = U - TS$

* For a steady flow process (1) \rightarrow (2) with $T_1 = T_2 = T_0$:

$$\rightarrow \dot{W}_{\max} = \dot{m}(b_1 - b_2) = \dot{m}[(h_1 - Ts_1) - (h_2 - Ts_2)] = \dot{m}(g_1 - g_2)$$

\rightarrow e.g. chemical changes like combustion!

REACTION PROPERTY CHANGES

* Basic representation: $\sum_i \nu_i A_i = 0$ where A_i are chemical species and ν_i are stoichiometric coefficients (products +ve)

* Then $\Delta \bar{H}_T^\circ = \sum_i \nu_i \bar{h}_i(T, p_0)$ and $\Delta \bar{G}_T^\circ = \sum_i \nu_i \bar{g}_i(T, p_0)$

$\rightarrow \bar{h}_i(T, p_0)$ is molar enthalpy at temperature T , standard pressure

\rightarrow each species supplied/withdrawn separately at T, p_0

$\rightarrow \Delta \bar{G}_T^\circ$ not tabulated, use $\Delta \bar{G}_T^\circ = -\bar{R}T \ln(K_p)$

$\rightarrow \Delta \bar{S}_T^\circ$ not tabulated either, use $\Delta \bar{G}_T^\circ = \Delta \bar{H}_T^\circ - T \Delta \bar{S}_T^\circ$

* For the case of combustion, $\Delta \bar{H}_{298K}^\circ = M_f [-LHV] = M_f [\Delta \bar{H}_{298K}^\circ]$

EFFICIENCY DEFINITIONS - for power plants

* $\eta_{\text{thermal}} = \dot{W}_{\text{net}} / \dot{Q}_{\text{in}} \equiv \eta_{\text{overall}} = \dot{W}_{\text{net}} / \dot{m}(-\Delta \bar{H}_{T_0}^\circ)$ if cyclic

* $\eta_{\text{thermal}} = \dot{W}_{\text{net}} / \dot{W}_{\max} = \dot{W}_{\text{net}} / \dot{E}_{\text{ex}} = \dot{W}_{\text{net}} / \dot{m}(-\Delta \bar{G}_{T_0}^\circ)$

IDEAL GAS MIXTURES

* Mole fractions $X_i = \frac{n_i}{n}$, partial pressures $p_i = \frac{n_i \bar{R}T}{V}$

\rightarrow then $p = \sum p_i$, $X_i = \frac{p_i}{p}$, $M = \sum X_i M_i$

* Mass fractions $Y_i = n_i M_i / \sum n_i M_i = \frac{M_i}{M} X_i$

* So $h = \sum Y_i h_i(T)$ and $\bar{h} = \sum X_i \bar{h}_i(T)$ etc.

* Use ideal gas result $\bar{s}_i(T, p_i) - \bar{s}_i(T, p) = -\bar{R} \ln\left(\frac{p_i}{p}\right) = -\bar{R} \ln(X_i)$

\rightarrow So $\bar{s} = \sum X_i \bar{s}_i(T, p_i) = \sum X_i \bar{s}_i(T, p) - \sum X_i \bar{R} \ln(X_i)$

3AS - Chemical Equilibrium

GIBBS EQUATION - multi-component systems, open to mass transfer

* Have $G = G(p, T, n_1, n_2, \dots, n_j, \dots, n_c)$

→ Differentiate $\Rightarrow dG = \left. \frac{\partial G}{\partial p} \right|_{T, n_j} dp + \left. \frac{\partial G}{\partial T} \right|_{p, n_j} dT + \sum_{i=1}^c \left. \frac{\partial G}{\partial n_i} \right|_{T, p, n_{i \neq j}} dn_i$

→ Use $\left. \frac{\partial G}{\partial p} \right|_{T, n_j} = V$ and $\left. \frac{\partial G}{\partial T} \right|_{p, n_j} = -S$ (closed system)

* So $dG = V dp - S dT + \sum_{i=1}^c \bar{\mu}_i dn_i$

CONDITION FOR EQUILIBRIUM

* At equilibrium at const. p and T get $dG = \sum_{i=1}^c \bar{\mu}_i dn_i = 0$

* Consider change in extent of reaction $d\xi = \frac{dn_1}{\nu_1} = \frac{dn_2}{\nu_2} = \dots = \frac{dn_i}{\nu_i}$

→ $\sum_{i=1}^c \nu_i \bar{\mu}_i d\xi = 0 \Rightarrow \sum_{i=1}^c \nu_i \bar{\mu}_i = 0$

* Now $\bar{\mu}_i(T, p_i) = \bar{h}_i(T) - T \bar{s}_i(T, p_i)$

→ $\bar{\mu}_i(T, p_i) = [\bar{h}_i(T) - T \bar{s}_i(T, p_0)] + T [\bar{s}_i(T, p_0) - \bar{s}_i(T, p_i)]$

→ apply ideal gas eqns: $\bar{\mu}_i(T, p_i) = \bar{\mu}_i^\circ(T) + RT \ln\left(\frac{p_i}{p_0}\right)$

* Combine: $\sum_{i=1}^c \nu_i \bar{\mu}_i^\circ(T) + \sum_{i=1}^c \nu_i RT \ln\left(\frac{p_i}{p_0}\right) = 0$

* Rearrange with $\sum_{i=1}^c \nu_i \bar{\mu}_i^\circ(T) = \Delta \bar{G}_T^\circ \Rightarrow \prod_{i=1}^c \left(\frac{p_i}{p_0}\right)^{\nu_i} = \exp\left(\frac{-\Delta \bar{G}_T^\circ}{RT}\right) = K_p(T)$

SOLVING EQUILIBRIUM PROBLEMS

- 1) Write out full chemical equations, LHS known, unknown numbers of moles a, b, \dots of all species on RHS
- 2) Conservation of atoms give some equations
- 3) Find partial pressures using mole fractions
- 4) Apply equilibrium equations, look up K_p at desired T
- 5) Solve simultaneously, may need to iterate.

EFFECT OF T, p , DISSOCIATION, ADIABATIC FLAMES

* Remember Le Chatelier for Chemistry.

* van 't Hoff: have $\Delta \bar{G}_T^\circ = \sum_{i=1}^c \nu_i \bar{g}_i(T, p_0) = -RT \ln(K_p)$

→ divide by RT , differentiate w.r.t T use $\left. \frac{\partial \bar{g}_i}{\partial T} \right|_p = -\bar{s}_i$ and $\sum_{i=1}^c \nu_i \bar{h}_i = \Delta \bar{H}_T^\circ$

→ so $\frac{d}{dT}(\ln(K_p)) = \frac{\Delta \bar{H}_T^\circ}{RT^2}$ or $\frac{d(\ln(K_p))}{d(1/T)} = \frac{\Delta \bar{H}_T^\circ}{R}$

* "Adiabatic flame T " is temperature of products after stoichiometric burning at 25°C with no heat loss.

* "degree of dissociation" is number of moles of e.g. CO from 1 mole CO_2 .

3A5 - Equations of State

MAXWELL'S RELATIONS

* Fundamental relations are $du = -pdv + Tds$ etc.

→ derive for dg, dh, dh by differentiating definitions, use Tds equation

* Can also write $du = \left. \frac{\partial u}{\partial v} \right|_s dv + \left. \frac{\partial u}{\partial s} \right|_v ds$

→ Comparing: $p = -\left. \frac{\partial u}{\partial v} \right|_s$, $T = \left. \frac{\partial u}{\partial s} \right|_v$ etc.

* $u = u(v, s)$ is a characteristic eqn of state, $p-v-T$ relation possible

* Differentiate again for Maxwell's relations: $\left. \frac{\partial p}{\partial s} \right|_v = -\frac{\partial^2 u}{\partial v \partial s} = -\frac{\partial^2 u}{\partial s \partial v} = -\left. \frac{\partial T}{\partial v} \right|_s$

* In the case of an ideal gas, can prove $h = h(T)$ only

→ use Maxwell relation $\left. \frac{\partial v}{\partial T} \right|_p = -\left. \frac{\partial s}{\partial p} \right|_T$ and $pv = RT$

→ $v = \left. \frac{\partial g}{\partial p} \right|_T = \left. \frac{\partial h}{\partial p} \right|_T - T \left. \frac{\partial s}{\partial p} \right|_T = \left. \frac{\partial h}{\partial p} \right|_T + T \left. \frac{\partial v}{\partial T} \right|_p \Rightarrow \left. \frac{\partial h}{\partial p} \right|_T = v - T \left. \frac{\partial v}{\partial T} \right|_p$

→ for ideal gas, RHS = 0 so h independent of p .

→ so $c_p = c_p(T)$, similar analysis of $f \Rightarrow u = u(T)$, $c_v = c_v(T)$

GAS DEFINITIONS

* Ideal gases - $pv = RT$ and $c_p - c_v = R$

* Perfect gases - c_p, c_v, δ constant

→ $h_2 - h_1 = c_p(T_2 - T_1)$, $s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$

* Semi-perfect gases - c_p, c_v, δ functions of T

→ $h_2 - h_1 = \int_1^2 c_p(T) dT$, $s_2 - s_1 = \int_1^2 \frac{c_p(T)}{T} dT - R \ln\left(\frac{p_2}{p_1}\right)$

* Imperfect - $pv \neq RT$, c_p and c_v fns(T, p), $c_p - c_v \neq R$

VAN DER WAALS EQUATION - modelling non-ideal behaviour.

* Two corrections: $(p + \Delta p)(V - \Delta V) = nRT$

→ ΔV is excluded volume, proportional to size of molecule,

→ Δp represents loss of pressure due to attractive forces at the container walls, reducing momentum transfer. Proportional to collision rate and number of neighbouring molecules, which are proportional to the number density of molecules

* Let $\Delta V = nb$, $\Delta p = a\left(\frac{n}{V}\right)^2$, $\bar{v} = V/n$

→ $(p + \frac{a}{\bar{v}^2})(\bar{v} - b) = RT$ or defining $p_R = \frac{p}{p_c}$, $T_R = \frac{T}{T_c}$, $V_R = \frac{\bar{v}}{\bar{v}_c}$:

$$\left(p_R + \frac{3}{V_R^2}\right)(3V_R - 1) = 8T_R$$

* Not very accurate, predicts $Z_c = \frac{p_c \bar{v}_c}{RT_c} = \frac{3}{8} = 0.375$

but actual values 0.222 - 0.333

3A5 - Fuel Cells

FUEL CELL TECHNOLOGY

- * Converts chemical energy directly into electricity - a steady flow process
- * V. low NO_2 and SO_2 emissions, electrochemical reaction is near-reversible so efficiency nearly 100%.
- * Types of fuel cell
 - Solid Oxide Fuel Cell - solid oxide electrolyte, charge carrier O^{2-} ion. Low ionic conductivity at low T requires that operating $T \sim 850 - 1000^\circ\text{C}$. This gives materials degradation but can use exhaust in combined cycle.
 - Proton Exchange Membrane Fuel Cell - solid polymer membrane electrolyte, charge carrier H_3O^+ ion. Requires platinum catalyst which is intolerant to CO . Operates at $80 - 100^\circ\text{C}$ so have to handle liquid water.

OPERATING PRINCIPLES - overall eqn. $\text{H}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{H}_2\text{O}$

- * Electric charge transfer during reaction drives electrical load.
- * All cells have anode and cathode separated by electrolyte.
- * Fuel supplied to anode, oxidant (air) to cathode.
 - Eg. in SOFC, at cathode - electrolyte interface $\frac{1}{2}\text{O}_2 + 2\text{e}^- \rightarrow \text{O}^{2-}$
 - O^{2-} ions migrate across electrolyte, then at anode interface have $\text{H}_2 \rightarrow 2\text{H}^+ + 2\text{e}^-$ and $2\text{H}^+ + \text{O}^{2-} \rightarrow \text{H}_2\text{O}$
- * Can reform the H_2 required at anode from methane
- * $\bar{W}_{E, \text{MAX}} = -\Delta\bar{G}_T^\circ$, heat rejected $\bar{Q} = -T\Delta\bar{S}_T^\circ$

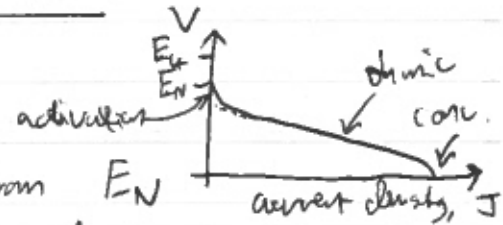
GIBBS AND NERNST POTENTIALS - $E_G = \frac{\bar{W}_{E, \text{MAX}}}{2F}$

- * Open circuit voltage of cell, ideally is $E_G = \frac{-\Delta\bar{G}_T^\circ}{2F}$, $F = q_e N_A$
- * In practice have $\bar{W}_{E, \text{MAX}} = -\Delta\bar{G}_T^P = -(\bar{g}_{\text{H}_2\text{O}}(T, p_{\text{H}_2\text{O}}) - \bar{g}_{\text{H}_2}(T, p_{\text{H}_2}) - \frac{1}{2}\bar{g}_{\text{O}_2}(T, p_{\text{O}_2}))$
- * $\bar{g}_i(T, p_i) = \bar{h}_i(T) - T\bar{s}_i(T, p_i) + T[\bar{s}_i(T, p_0) - \bar{s}_i(T, p_i)] = \bar{g}_i(T, p_0) + \bar{R}T \ln\left(\frac{p_i}{p_0}\right)$
- * With operating pressure p_i , $\bar{g}_i(T, p_i) = \bar{g}_i(T, p_0) + \bar{R}T \ln(x_i) + \bar{R}T \ln\left(\frac{p_i}{p_0}\right)$
 - so $\bar{W}_{E, \text{MAX}} = -\Delta\bar{G}_T^\circ + \bar{R}T \left[\ln\left(\frac{x_{\text{H}_2} \sqrt{x_{\text{O}_2}}}{x_{\text{H}_2\text{O}}}\right) + \frac{1}{2} \ln\left(\frac{p_i}{p_0}\right) \right]$
 - then $E_N = \bar{W}_{E, \text{MAX}} / 2F = E_G + \frac{\bar{R}T}{2F} \left[\ln\left(\frac{x_{\text{H}_2} \sqrt{x_{\text{O}_2}}}{x_{\text{H}_2\text{O}}}\right) + \frac{1}{2} \ln\left(\frac{p_i}{p_0}\right) \right]$

3A5 - Fuel Cell Losses, Phase Equilibrium

FUEL CELL LOSSES

- * Under load, terminal voltage less than E_N
- Activation loss - overpotential required to reach activation energy to get desired reaction rate
- Ohmic loss - linear, resistance to ions in electrolyte and electrons through electrodes and connectors
- Concentration loss - caused by difference in X_{H_2} between fuel inps and anode interface. Need difference to drive diffusive flux at desired rate.



EQUILIBRIUM CRITERIA

- * Take Gibbs definition, $G = H - TS$ and differentiate:
 - $dG = dH - TdS - SdT$, also use $TdS = dH - Vdp$
 - $dG = Vdp - SdT$, $dG = vdp - sdT$ (between equilibrium states)
- * 1st Law: $dU = dQ - dW = dQ - pdV$, 2nd Law: $dS = \frac{dQ}{T} + dS_{in}$
 - Combine $\Rightarrow dU + pdV - TdS = -TdS_{in} \leq 0$
 - i.e. if V and S const., spontaneous changes decrease U
 - eliminate dU to find if p and T const. G minimum at equilibrium
- * Consider phases α, β at const. T and p at equilibrium
 - Must have $T_\alpha = T_\beta = T$ or heat transfer would occur
 - Must have $p_\alpha = p_\beta = p$ or momentum transfer would occur
 - $G = m_\alpha g_\alpha + m_\beta g_\beta$, $dG = m_\alpha dg_\alpha + m_\beta dg_\beta + g_\alpha dm_\alpha + g_\beta dm_\beta$
 - but from above $dg_i = v_i dp_i + s_i dT_i = v_i dp_i + s_i dT_i = 0$
 - total mass const. $\Rightarrow dm_\alpha + dm_\beta = 0$, at equilibrium:
 - $dG = (g_\alpha - g_\beta) dm_\alpha = 0$, dm_α arbitrary so $\underline{g_\alpha = g_\beta}$ or $\underline{p_\alpha = p_\beta}$

CLAUSIUS - CLAPEYRON EQUATION

- * Along saturation line $p_f = p_g \Rightarrow h_f - Ts_f = h_g - Ts_g \Rightarrow s_f = \frac{h_f - h_g}{T}$
- * Also $ds_f = ds_g$, with $dg = vdp - sT \Rightarrow v_f dp_f - s_f dT = v_g dp_g - s_g dT$
- * Rearranging: $\underline{\frac{dp_s}{dT} = \frac{s_f - s_g}{v_f - v_g} = \frac{h_{fg}}{v_{fg} T}}$
- * V. useful approximations if $T \ll T_{crit}$: $\underline{v_g \gg v_f}$, $\underline{p_s v_g = RT}$

3A5 - POWER GENERATION - Background and Future Trends

ATMOSPHERIC CARBON

- * Need to reduce emissions, power generation has large contribution.
- * C emission in $\text{kg}/\text{J} = f_c / \eta \Delta H_{298}^\circ = f_c / \eta \text{LCV}$
 - i.e. for given fuel can ~~only~~ reduce C by increasing efficiency (or choose a different fuel)
- * Alternative is C capture and storage - demonstrated but not viable.
 - Post-combustion: remove CO_2 from exhaust by "scrubbing" with amine solution. Retrofittable. Poor efficiency because $[\text{CO}_2]$ low.
 - Pre-combustion: reform fuel to H_2 and CO_2 using steam reforming and shift reactions. $[\text{CO}_2]$ higher so easier to remove coal.
 - Must store CO_2 for 100s of years - long-term uncertainty. Can use empty gas/oil fields, aquifers, oceans.
- * Zero C generation: nuclear or renewables. Need fast-start capacity to cope with fluctuating e.g. wind capacity.

GENERAL

- * Gas Turbines: efficiency around 40%. Used for grid generation, remote and standby power generation.
 - High pressure-ratio aeroderivatives can have two shafts!
- * Steam Turbines: largest man-made power source worldwide. Based on Rankine cycle. Use heat from fossil fuels, nuclear reactors, exhaust of GT in combined cycle. $\eta \sim 45\%$.

FUTURE TRENDS

- * Fuel prices increasing, pressure to reduce emissions
 - More C capture focus on fast-response capacity as outlined above to reduce emissions.
 - Continued effort to increase cycle efficiencies and reduce energy losses.
- * Steam Injected Gas Turbine - STIG - into combustion to reduce peak temperature and hence NO_x , increase power.
 - can give 10% efficiency, more mass in turbine \Rightarrow power gains.
 - exhaust used to heat the steam, reduce stack temperature.
 - but still have mixing and heat transfer losses.
- * Evaporative GT Cycles: add water to make compression more isothermal as water evaporates \Rightarrow less compressor work. Also reduces compressor exit T so reduces cooling requirement.
 - But must redesign compressor aerodynamics, small evaporation.
- * Fuel Cell GT Cycles: reduces combustion irreversibilities as converts fuel directly to electricity. Then burn residual fuel and use exhaust at high T to power turbine.

3AS - POWER GENERATION - Gas Turbines I

COMPRESSOR - BURNER - TURBINE GT - Joule Cycle

- * closed air-standard Joule cycle is model of open-circuit CBT G
 - Assume all processes reversible, no pressure drop in heat exchanger
 - Also c_p and γ constant air values around cycle.
 - No time for heat transfer in compressor/turbine \Rightarrow adiabatic
- * For Joule cycle, η_c increases monotonically with $\uparrow r_p$
 - High work ratio (less susceptibility to irreversibility) requires high overall temperature ratio, low pressure ratio.
- * In a real gas turbine (compared to Joule cycle):
 - have non-isentropic turbomachinery, pressure drop in combustor
 - c_p, γ functions of T , c_p greater for combustion products
 - Combustion occurs internally, added fuel increases w

ISENTROPIC AND POLYTROPIC EFFICIENCIES - η_{is} and $\eta_{p,t}$

- * Isentropic is defined in terms of theoretical isentropic work
 - $\eta_{is,c} = \frac{\text{ideal } w_x}{\text{actual } w_x} = \frac{h_{2s} - h_1}{h_2 - h_1}$, $\eta_{is,t} = \frac{\text{actual } w_x}{\text{ideal } w_x} = \frac{h_3 - h_4}{h_3 - h_{4s}}$

- * Non-isentropic turbomachinery gives $\eta_c \uparrow$ with $\uparrow T_3$
 - Also cannot maximise η_c and w_x simultaneously.
 - Pressure ratio for max $\eta_c >$ for max w_x

- * Polytropic efficiencies based on small stages

→ $\eta_{p,c} = \frac{dh_s}{dh}$, $\eta_{p,t} = \frac{dh}{dh_s}$, use $T ds = dh - v dp = dh - RT \frac{dp}{p}$

→ so for $ds=0$, $dh_s = RT \frac{dp}{p}$, integrate $\Rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$, $\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$

INTERCOOLING

- * Splits compression into two stages, cools flow between low and high pressure sections
- * Makes compression more isothermal \Rightarrow less work needed.
- * Assume intercooler exit T equal to first compressor inlet T .
- * Consider intercooling as an addition to Joule cycle
 - More area inside $T-s$ curve \Rightarrow greater specific work
 - Efficiency reduces because average pressure ratio reduces.
- * Another benefit is lower turbine inlet $T \Rightarrow$ reduces cooling required.
- * Can select intercooler pressure to optimise η
 - Find that across first compressor $r_{p1,opt} = \sqrt{r_{p,ov}}$

3A5 - POWER GENERATION - Gas Turbines II

REHEATING

- * Similar to intercooling - isothermal expansion makes more work.
- * Split expansion into HP and LP turbines, increase flow temperature in between using a 2nd combustion stage.
- * Assume reheat occurs at constant pressure, exit T of second combustor equal to first turbine inlet T.
- * By the "exon cycle" argument used for intercooling:
 - Reduces cycle efficiency
 - Increases number of high T components, hence cost.
 - Increases work output, can optimize pressure ratios as before.

REUPERATION

- * Requires turbine exit T > compressor exit T
 - i.e. lower pressure ratio, high temperature ratio T_3/T_1 .
- * Uses a (big, expensive) heat exchanger to preheat combustor air.
- * Recovers energy in exhaust so reduces in and increases η .
- * Heat exchanger effectiveness: $\epsilon = \frac{Q_{actual}}{Q_{max}} = \frac{(m\dot{c}_p)_h (T_{hi} - T_{ho})}{(m\dot{c}_p)_{max} (T_{hi} - T_{ci})} \leq 0.8$ usually.
- * For modelling, assume no pressure losses, $\epsilon = 1$
 - Ideal cycle analysis \Rightarrow max η at zero pressure ratio!
 - In reality η_{max} occurs at lower r_p than for simple GT but not 0.
 - In reality, $\epsilon < 1$ so more fuel required to reach given combustor outlet T, cycle efficiency changes, reduces.
 - Also pressure losses reduce r_p and hence w_n and η .

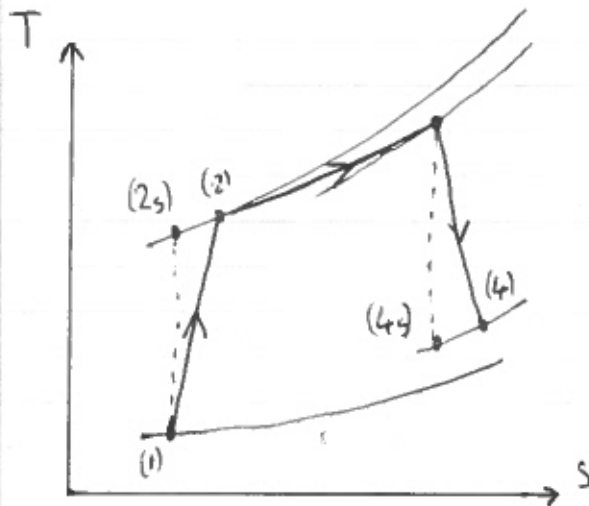
BLADE COOLING - allows higher turbine inlet T, greater w_n and η

- * 15-25% of compressor flow used to cool blades in combustor.
 - But creates heat transfer and mixing irreversibilities.
- * In air-standard cycle analysis, additional assumptions
 - Already solved heat transfer, know fraction of cooling flow needed, m_c .
 - Cooling flow choked to give correct injection pressure
 - Mixing loss small relative to thermodynamic loss.
 - No pressure loss in combustor, $c_p = \text{const}$, isentropic combustion.
- * To analyse, be careful with varying mass flow rates.
- * Use SPICE for mixing process.

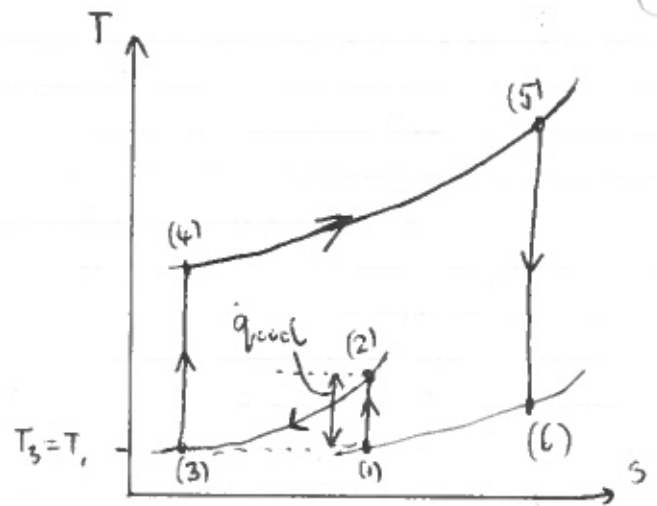
ENERGY ANALYSIS

- * Big loss due to exhaust gas, because of high T
- * Big loss in combustor, reheat because chemical reaction proceeds far from equilibrium - not reversible.
- * Cooling loss same order as uncooled, in isentropic expansion.

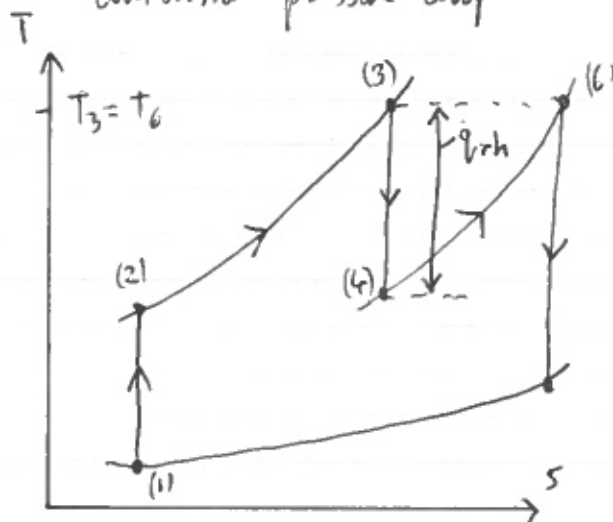
3AS - POWER GENERATION - Gas Turbine Cycle Diagrams



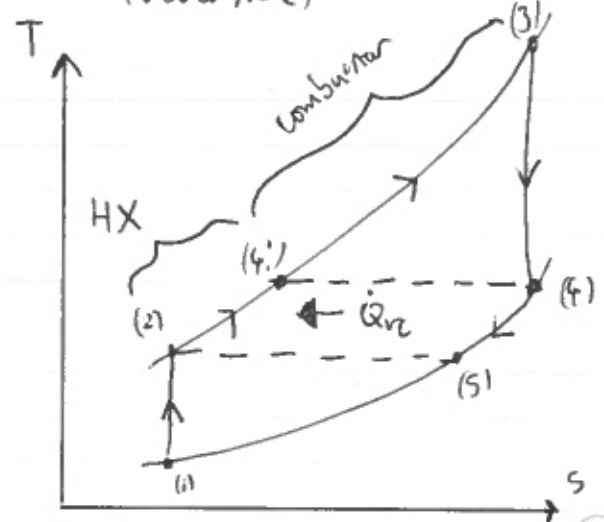
Joule cycle with non-isentropic turbomachinery, combustor pressure drop



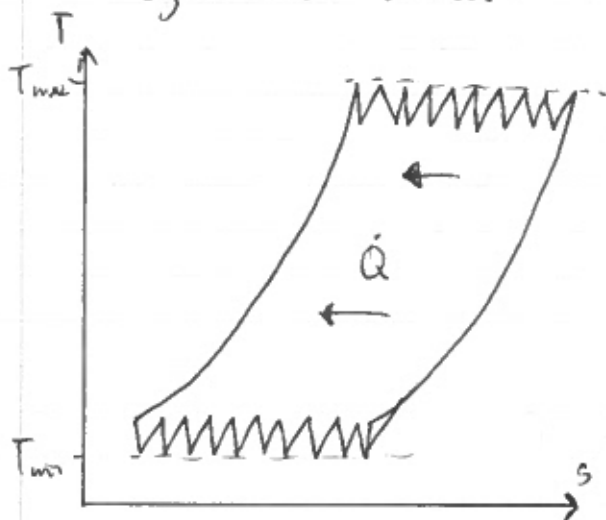
Intercooled air-standard cycle (reversible)



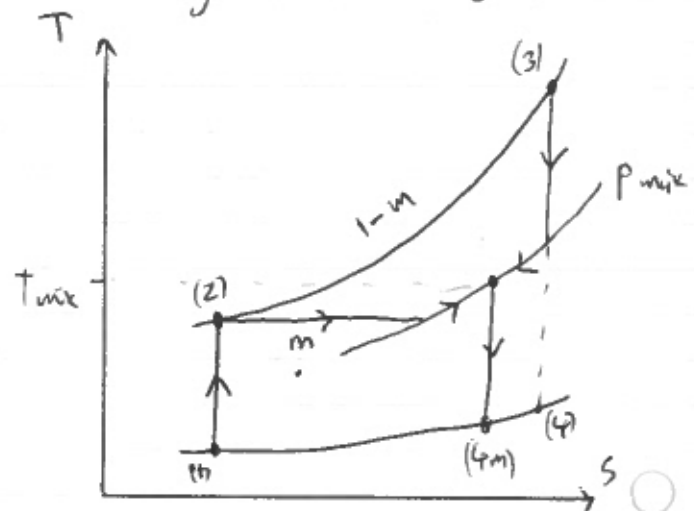
Reversible air-standard cycle with reheat.



Reversible recuperated cycle with $E=1$



Intercool, reheat, recuperate tends to Carnot cycle!



Air-standard cycle with blade cooling flow

3A5 - POWER GENERATION - Steam Cycles I

RANKINE CYCLE - a true thermodynamic cycle

- * Feed pump compresses liquid water, constant pressure boiler and superheater, adiabatic turbine, constant pressure condenser.
- * Useful approximation for feed pump work:
$$w_p = h_2 - h_1 = (h_{2s} - h_1) / \eta_p = \frac{1}{\eta_p} \int_1^{2s} \frac{dp}{\rho} \approx \frac{P_2 - P_1}{\eta_p \rho} \quad (\text{using } T_{d_2} = dh - v dp)$$
- * Little work needed to compress water, i.e. $w_T \gg w_p$
- * Can improve cycle efficiency, in general, by raising average T of heat addition or lowering average T of heat rejection.
 - Or for an irreversible cycle, reduce lost work.
 - Condenser temperature constant and close to ambient
- * So focus on main heat reception T , T_{in}
 - e.g. raising boiler pressure - for fixed T_{max} raises T_{in} until supercritical gives diminishing returns in η .

STEAM REHEAT

- * Increasing boiler pressure reduces dryness fraction at turbine outlet, bad for blades, erosion etc.
- * Counteract by splitting expansion into two stages and returning steam to boiler in between.
 - This increases dryness fraction and specific work.
 - Can also improve T_{in} and efficiency.
- * High reheat p gives large T_{in} but small extent Δ , and vice versa
 - Optimum is approx $1/4$ of main boiler p .

FEEDHEATING - preheating water before boiler

- * Small amount of steam extracted from turbine, mixed with feed water at constant pressure.
 - Extracted flow fraction is such that final state saturated water at a higher temperature.
 - i.e. Feed water T increased without external heating. increases T_{in} , net increase in η despite irreversible mixing.
- * Note that a pump is required for each feed heater to raise water pressure to steam extraction pressure.
- * Use SFE to find final mixed state of water and steam.
- * Indirect contact feedheating: uses heat exchangers instead of mixing to reduce irreversibilities.
- * Maximum benefit obtained by using infinite number of reversible heaters - in practice use 6-9.

3A5 - POWER GENERATION - Steam Cycles II

COMBUSTION AND EFFICIENCIES

- * For burning at standard conditions, $Q_{ideal} = \dot{m}_f [-\Delta H_o]$
- * In reality, reactants enter at ambient conditions and leave at a much higher stack temperature T_x
- This reduces Q_{in} compared to constant T combustion.
- Using SFEE: $Q_{in} = \dot{m}_f [-\Delta H_o] - (\dot{m}_a + \dot{m}_f)(h_{px} - h_{po})$

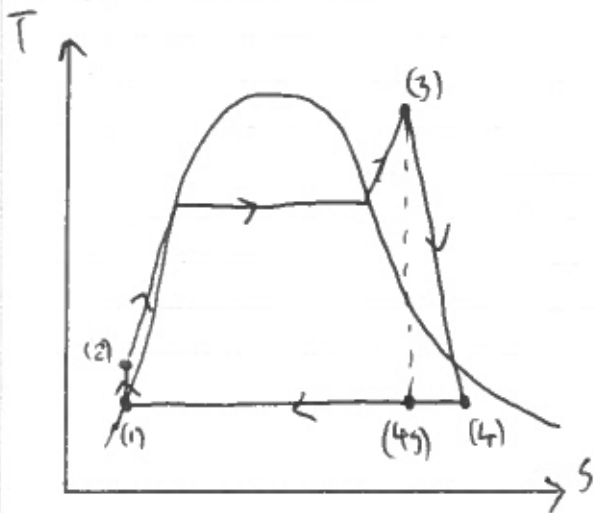
* So boiler efficiency: $\eta_b = \frac{Q_{in}}{\dot{m}_f [-\Delta H_o]} = 1 - \frac{(A+1)(h_{px} - h_{po})}{-\Delta H_o}$ ($A = AFR$)

- * Now considering complete plant, overall efficiency:

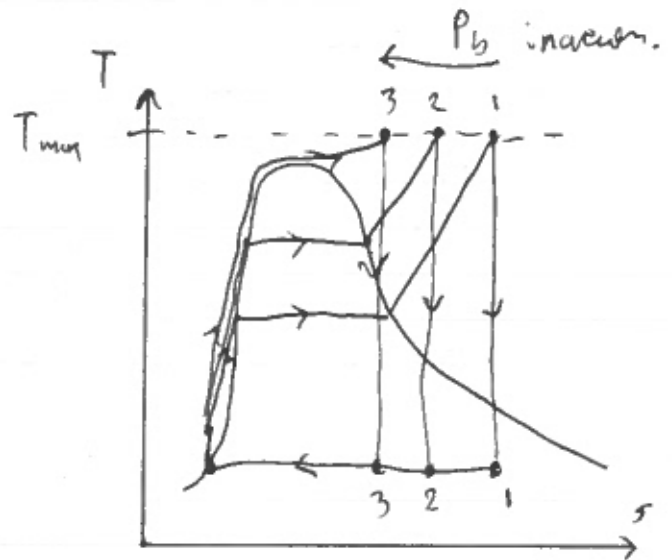
$$\eta_{ov} = \frac{W_{net}}{\dot{m}_f [-\Delta H_o]} = \frac{W_{net}}{Q_{in}} \times \frac{Q_{in}}{\dot{m}_f [-\Delta H_o]} = \eta_{cy} \times \eta_b$$

- * To maximize η_{ov} and η_b for given cycle minimise T_x
- But to avoid corrosion can't let water vapour condense in flue $\Rightarrow T_x > T_{dew}$.

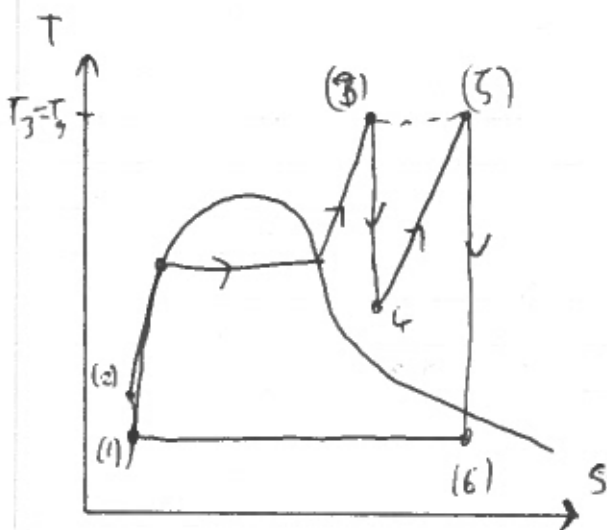
CYCLE DIAGRAMS:



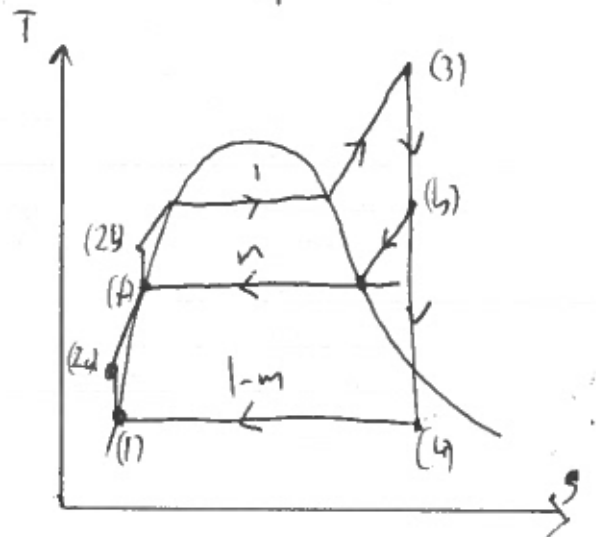
Basic Rankine Cycle



Effect of boiler pressure
3 is supercritical.



Single reheat cycle

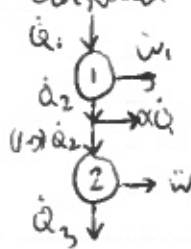


Single feedwater

3AS - POWER GENERATION - Combined Cycles

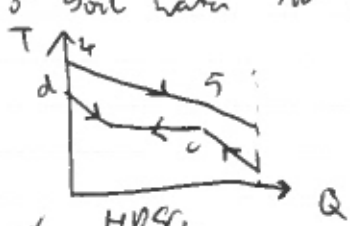
GENERAL ANALYSIS

- * For steady flow devices - minimum temperature $\approx T_{amb} + 10K$, max delivery temperature from boiler/combustor $\approx 1800K$ (material limit)
 - So η_{max} is $1 - T_{min}/T_{max}$ as for Carnot. $\gg \eta_{GT}$ or η_{ST} .
 - We combine cycles with narrower temperature ranges.
- * Would ideally combine two Carnot engines but these don't exist.
 - First cycles used Rankine using topping cycle, steam bottoming.
 - In practice use GT topping, ST bottoming.
- * Consider combination of two cycles with efficiencies η_1 and η_2
 - Fraction of heat α lost between cycles.
 - $\dot{w} = \dot{w}_1 + \dot{w}_2 = \eta_1 \dot{Q}_1 + \eta_2 \dot{Q}_2 (1 - \alpha) = \eta_1 \dot{Q}_1 + \eta_2 \dot{Q}_1 (1 - \eta_1) (1 - \alpha)$
 - * Then $\eta_{cc} = \dot{w} / \dot{Q}_1$, let $(1 - \alpha) = \eta_b$
 - $\eta_{cc} = \eta_1 + \eta_b \eta_2 (1 - \eta_1) = \eta_b \eta_2 + \eta_1 (1 - \eta_b \eta_2)$
 - where $\eta_b \eta_2$ is efficiency of lower plant



COMBINED STs and GTs

- * Can achieve $\sim 60\%$ overall efficiency.
- * Hot GT exhaust fed into HRSG to boil water for steam cycle.
 - Represent HRSG on T-Q diagram:
 - Ratio of mass flows fixed by lower labelled slopes.
 - Pinch point $\Delta T_p = T_5 - T_c$ controls rate of heat transfer hence size of HRSG.
- * Optimisation: overall efficiency is $\eta_{cc} = \frac{\dot{W}_{GT} + \dot{W}_{ST}}{\dot{m} \Delta H_0}$
 - \dot{m} and \dot{W}_{GT} fixed by GT \Rightarrow to maximise η_{cc} need to maximise \dot{W}_{ST} . Do this by changing steam pressure.
 - As pressure reduced more heat transferred from GT exhaust \Rightarrow exhaust T falls, exhaust energy loss reduces
 - But this increases area between gas/steam lines \Rightarrow HRSG costs increase.



FEEDHEATING AND PREHEATING

- * Feedheating increases steam HRSG inlet T so heat transferred above pinch point reduced
- Exhaust T high, greater energy loss \Rightarrow not used.

